1. A chicken lays n eggs. Each egg independently does or doesn’t hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn’t survive (independently of the other eggs), with probability s of survival. Let N ⇠ Bin(n, p) be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don’t survive (so X + Y = N). Find the marginal PMF of X, and the joint PMF of X and Y . Are they independent?

Ans.

the probability that a egg hatches and chick survives is psps. We can consider each egg as a Bernoulli trial each with a success (hatching and surviving) probability psps. There are nn independent trials, so X∼Bin(n,ps)X∼Bin(n,ps).

But I am trying to prove this more rigorously.

For any 1≤i≤n1≤i≤n we have

P(X=i)=∑j=0nP(X=i|N=j)P(N=j)=∑j=inP(X=i|N=j)P(N=j)=∑j=in(ji)si(1−s)j−i(nj)pj(1−p)n−j

the probability that a egg hatches and chick survives is psps. We can consider each egg as a Bernoulli trial each with a success (hatching and surviving) probability psps. There are nn independent trials, so X∼Bin(n,ps)X∼Bin(n,ps).

But I am trying to prove this more rigorously.

For any 1≤i≤n1≤i≤n we have

P(X=i)=∑j=0nP(X=i|N=j)P(N=j)=∑j=inP(X=i|N=j)P(N=j)=∑j=in(ji)si(1−s)j−i(nj)pj(1−p)n−j